

Fractile vs. Equal



FRED® Map
APPLICATION EXERCISE

Lesson Authors

Diego Mendez-Carbajo, PhD, Federal Reserve Bank of St. Louis

Mary Suiter, PhD, Federal Reserve Bank of St. Louis

Standards and Benchmarks (see page 15)

Description

Students work with data that represent the ages of 24 people to learn the difference between categorizing data into fractile intervals and equal intervals. Students discuss dividing bonus points among class members to understand what per capita means. Students then look at per capita personal income by state using a FRED® map. They compare per capita personal income displayed with data in equal intervals and with data in fractile intervals.

Economic and Statistical Concepts

Equal intervals: Data ranges of equally sized intervals.

Fractile intervals: Data ranges with the same number of data points per interval.

Income: Payment people earn for the work they do.

Per capita personal income: The total income per person in a region determined by dividing the total income by the total population.

Objectives

Students will be able to

- explain the differences between equal intervals and fractile intervals,
- define personal income and per capita personal income; and
- compare tables and maps of the same data displayed with equal intervals and with fractile intervals.

Grade Level

Grades 7-10

Compelling Question

How do fractile and equal intervals create different data categories?

Time Required

45-60 minutes

Materials

- Visuals of Handouts 1 and 2
 - Handout 1, one copy for each pair of students
 - Handouts 2 and 3, one copy of each for each student
 - Handouts 2 and 3 Answer Keys, one copy of each for the teacher
 - Scissors for each pair of students
 - A calculator for each student
-

Preparation

- Make sure you are comfortable navigating around FRED® and finding values for data series used in this lesson. A demonstration is located at <https://fredhelp.stlouisfed.org/>.
 - Go to <https://www.stlouisfed.org/education/economic-lowdown-video-series/episode-7-gross-domestic-product> to review the definitions for gross domestic product and income.
 - Go to <https://fredblog.stlouisfed.org/2022/08/a-lesson-in-mapping-population-data/> to see a short primer on fractile and equal intervals in FRED® maps.
-

Procedure

1. Explain that a marketing research firm interviewed several people at a large suburban mall regarding their shopping habits. During the interviews, they asked each participant to provide their age.
2. Display a visual of *Handout 1: Age Cards* and explain that this visual lists the ages of one group of participants that was interviewed. Explain that students will use the age information to learn about ways to organize data to make them easy to understand and analyze. Point out that there are at least two ways to organize this list of ages or set of data. One way is to establish categories or age-range values and to then place each age into the appropriate category—for example, 20-25 years, 26-31 years, etc. Each category has the same age range—that is, six years.

Another possibility is to place an equal, or nearly equal, number of data points/ages in each category and to then build the ranges accordingly. For example, suppose there are five people in the 16-19 age range, five people in the 20-30 range, five people in the 31-39 range, etc. In this case, the range for the categories varies, but the quantity of ages in each category is the same for all categories.

3. Divide the class into student pairs. Distribute a copy of *Handout 1: Age Cards* and a pair of scissors to each pair of students. Tell half of the pairs in the room to cut out their cards and organize the data into categories, with each category having the same number of age cards/data points. Tell these student pairs to write the value of the beginning and ending range for each category.

For example, if each range is supposed to have three data points/ages in it, then one category might be 15-19 years. With this type of categorizing, the range in number of years per category might vary from category to category, but there will be the same number of data points, or cards, in each category. Point out that the groupings must have consecutive ages—that is, 15-19, 20-27, etc., versus 15, 20, 27, etc. Point out that there are different possible answers.

4. Tell pairs in the other half of the class to cut the cards apart and divide the cards into categories of equal range. Tell them to write the range for each category, such as 20-25 years, etc. Remind them that each age-range/category should be equal—that is, it should include the same number of years. For example, a category titled “20-27 years” would include eight ages (20, 21, 22, 23, 24, 25, 26, 27) and therefore other ranges should include eight ages, too. Point out that there are different possible answers.
5. Allow time for students to work. Ask several pairs from each half of the class to share their work.
6. Display *Handout 2: Equal vs. Fractile*, refer to Table A, and review the categories/intervals in column 1. Ask students if the age range (number of ages) for each category is the same. (*Yes, five ages*)
7. Distribute a copy of Handout 2 to each student and have students look at Table A. Review the intervals in column 1 and instruct students to organize their cards into those intervals. Discuss the following:
 - The first interval is 15-19 years.
 - How many age cards fall in this interval? (*Three—16, 17, and 19*)
 - Write these numbers in the second column of Table A, across from the 15-19-year interval.
 - Continue with similar discussions for each interval in Table A. Refer to *Handout 2: Equal vs. Fractile—Answer Key* to check student work and record information in the chart.
8. Refer to Table B on the visual of Handout 2 and review the intervals in column 3. Instruct student pairs to organize their cards into these intervals. Ask students to report the number of age cards in each category. Write that information in column 4, across from each interval. Refer to *Handout 2: Equal vs. Fractile—Answer Key*. Discuss the following:
 - How do the age intervals differ in Table A and Table B? (*In Table A, column 1, each age interval is the same size. In Table B, column 3, the age intervals are different sizes.*)
 - How do the number of data points in each age interval differ in Table A and Table B? (*In Table A, there is a different number of data points in each age interval. In Table B, there is the same number of data points in each age interval.*)

9. Explain that when data are divided into equal-size ranges or intervals, the method of categorizing the data is called **equal interval**. In the first row of Table A, next to the words “Table A,” write the title “Equal Intervals.” Then explain that when data are divided into intervals so that there is the same number, or nearly the same number, of data points in each interval, the method of categorizing the data is called **fractile interval**. In the first row of Table B, next to the words “Table B,” write the title “Fractile Intervals.”
10. Ask students where they would have placed one additional card representing a 20-year-old person. Explain that the additional 20-year-old would have fallen into the “15-20 years” interval, giving that interval five data points.
11. Point out that with both equal intervals and fractile intervals, the set of cards or data points were the same, but the way in which the data were organized was different. Point out that neither way of organizing the data—equal interval or fractile interval—is wrong or right. But the organization may give those reviewing the data different ideas about the data and allow people to tell slightly different stories about the data.
12. Tell students they are going to look at some data in FRED®. Explain that FRED® is a database that can display data on maps. In this case, students will look at per capita personal income data. Define “per capita” as per person. To help students understand the idea of per capita, use the following example: (*NOTE: For this example, determine the total number of bonus points available by multiplying the number of students in your class by 10.*)
 - Over the semester so far, students have earned a total of 240 bonus points.
 - To determine how the bonus points relate to the number of students in the class, we need to determine per capita bonus points.
 - The same number of points will be available for each student in the class. How many points will be available per student? (*Number of points ÷ Number of students = 10 points*)
13. Open an internet browser and navigate to: <https://fred.stlouisfed.org/>. Tell students that this is the FRED® site of the Federal Reserve Bank of St. Louis. FRED® can show data as line graphs and in maps.
14. Tell students that one set of data available in FRED® is per capita personal income by state. Remind students that per capita means per person. Explain that income is the payment people earn for the work they do. **Per capita personal income** is the total income earned by individuals in a state, region, or country during a year, divided by the population of the state, region, or country. For this map, we will use per capita personal income by state.
15. Use the search box to look for and select “Per Capita Personal Income in Michigan” (Dollars, Annual, Not Seasonally Adjusted). The website will display this line graph: <https://fred.stlouisfed.org/graph/?g=16d9l>.

16. Click on the green button “View Map.” The website will show a map of the United States with the latest available data on per capita personal income in each state: <https://fred.stlouisfed.org/graph/?m=16fNq>.
17. Explain that FRED® maps allow users to organize data into fractile categories. Discuss the following:
- What does it mean to organize data into fractile categories? (*The sizes of the intervals may vary, but the number of data points—in this case, states—in each interval will be equal.*)
18. Move the cursor over each state to see that state’s per capita personal income. Refer students to the map’s legend/key. Point out that the data are reported in dollars. Discuss the following:
- Which colors represent higher per capita incomes? (*The darker colors*)
 - Which colors represent lower per capita incomes? (*The lighter colors*)
 - Referring to the legend shown at the bottom of the map, what is the interval for each category? (*To find each interval, calculate the difference between each pair of consecutive cutoff values.*)
 - Are the category intervals for this map equal in size? (*The intervals are not equal.*)
 - How many states are in the lowest category of personal income, and which states are included? (*Count and name the light-colored states; there should be 10 or 11.*)
 - How many states are in the highest category of personal income, and which states are included? (*Count and name the dark-colored states; there should be 9 or 10.*)
 - Zooming in on the Northeast region, which state has the highest per capita personal income in this region? (*Compare Connecticut and Massachusetts with the other states in the region.*)
 - What is the per capita personal income in the District of Columbia? (*Allow students to guess why they are not able to locate the state with this level of personal income. Zoom in around the Washington, DC, area to reveal that the highest per capita personal income is earned in the District of Columbia.*)
 - What would explain the variance between the differences in the first four categories and the difference between the 4th and 5th category? (*The high level of per capita personal income in Washington, DC.*)
 - What might you summarize about state-level per capita income in the United States from looking at this map? (*Answers will vary but may include that states colored similarly have very different per capita incomes.*)
19. Explain that FRED® maps also allow users to organize data into equal interval categories. Discuss the following:
- What does it mean to organize data into equal interval categories? (*The sizes of the intervals will be almost equal, but the number of data points—in this case, states—in each interval will be different.*)

20. Click on “Edit Map.” Use the dropdown menu next to “Data grouped by:” to select “Equal Interval.” Discuss the following:
- Into how many intervals are the data divided? (*Five*)
 - Are the intervals equal? (*Yes; calculate the difference between each pair of consecutive cutoff values.*)
 - Are the data categorized into equal intervals or fractile intervals? (*Equal intervals*)
 - Which color represents the highest category of per capita personal income? (*The darkest color*)
 - Which color represents the lowest category of per capita personal income? (*The lightest color*)
 - Would you describe this map as primarily light-colored or dark-colored? (*Primarily light-colored*)
 - If the map is primarily light-colored, what might you summarize about state-level per capita incomes in the United States? (*Answers may vary. Some students might say that per capita incomes in the United States are relatively low and homogeneous or similar.*)
 - What is different about the number of data points (states) in each interval for this map? (*The intervals are equal in size, and the number of data points in each interval vary.*)
21. Point out that if people do not carefully review the keys for these two maps (*equal interval and fractile interval*), the information might be misinterpreted. It is important that people ask questions about data that are presented to them, such as how the data are organized and why the data are organized in that way.

Closure

22. Review important content of the lesson by discussing the following:
- How does categorizing data into equal intervals differ from categorizing data into fractile intervals? (*With equal categorization, intervals or ranges of the same size are established and data are categorized in those ranges. There will be varying numbers of data points in each range. With fractile categorization, intervals or ranges are established so that there is an equal number, or nearly equal number, of data points in each range or interval.*)
 - What does per capita mean? (*Per person*)
 - What is personal income? (*The payment people earn for the work they do*)
 - What is per capita personal income? (*The total income earned by people in a state, region, or country, divided by the population of the state, region, or country*)

Assessment

23. Distribute a copy of *Handout 3: Assessment* to each student. Go to <https://fred.stlouisfed.org/>. Use the search box to look for and select “Gross Domestic Product: All Industry Total in Michigan” (Millions of Dollars, Quarterly, Seasonally Adjusted Annual Rate). The website will display this line graph: <https://fred.stlouisfed.org/graph/?g=16fOy>.
24. Click on the green button “View Map.” The website will show a map of the United States with the latest available data on gross domestic product in each state: <https://fred.stlouisfed.org/graph/?m=16fOI>.
25. Refer students to the map, as well as the map legend, and ask the questions in Part I of Handout 3. After you ask each question, allow time for students to write their responses on the handout. Refer to *Handout 3: Assessment—Answer Key* to check student responses for Part I.
26. Click on “Edit Map” and change the “Data grouped by:” to “Equal Interval.” Refer students to Part II of Handout 3. Read each question and allow time for students to record their answers on the handout. Refer to *Handout 3: Assessment—Answer Key* to check student responses for Part II.

Handout 1: Age Cards

19 years	20 years	21 years	22 years
16 years	17 years	22 years	23 years
30 years	31 years	34 years	42 years
43 years	44 years	45 years	46 years
50 years	51 years	52 years	62 years
49 years	37 years	48 years	67 years

Handout 2: Equal vs. Fractile

Table A		Table B	
Category/interval	Number of cards/ data points	Category/interval	Number of cards/ data points
15-19 years		15-20 years	
20-24 years		21-29 years	
25-29 years		30-39 years	
30-34 years		40-45 years	
35-39 years		46-50 years	
40-44 years		51-69 years	
45-49 years			
50-54 years			
55-59 years			
60-64 years			
65-69 years			

Handout 2: Equal vs. Fractile—Answer Key

Table A		Table B	
Category/interval	Number of cards/ data points	Category/interval	Number of cards/ data points
15-19 years	<i>16, 17, 19</i>	15-20 years	<i>16, 17, 19, 20</i>
20-24 years	<i>20, 21, 22, 22, 23</i>	21-29 years	<i>21, 22, 22, 23</i>
25-29 years		30-39 years	<i>30, 31, 34, 37</i>
30-34 years	<i>30, 31, 34</i>	40-45 years	<i>42, 43, 44, 45</i>
35-39 years	<i>37</i>	46-50 years	<i>46, 48, 49, 50</i>
40-44 years	<i>42, 43, 44</i>	51-69 years	<i>51, 52, 62, 67</i>
45-49 years	<i>45, 46, 48, 49</i>		
50-54 years	<i>50, 51, 52</i>		
55-59 years			
60-64 years	<i>62</i>		
65-69 years	<i>67</i>		

Handout 3: Assessment (page 1 of 2)

Part I:

Directions: Refer to the FRED® map displayed in class to answer each question below.

1. In what units are the data reported in the map key?
2. How many categories of data are there?
3. What is the interval for each category?
4. Which color represents the highest amount of gross domestic product by state?
5. Which color represents the lowest amount of gross domestic product by state?
6. Based on this map, does there appear to be much difference from state to state?
7. Why is this an example of fractile categorization?

Handout 3: Assessment—Answer Key (page 1 of 2)

Part I:

Directions: Refer to the FRED® map displayed in class to answer each question below.

1. In what units are the data reported in the map key?
Millions of dollars
2. How many categories of data are there?
Five
3. What is the interval for each category?
Calculate the difference between each pair of consecutive cutoff values.
4. Which color represents the highest amount of gross domestic product by state?
The darkest color
5. Which color represents the lowest amount of gross domestic product by state?
The lightest color
6. Based on this map, does there appear to be much difference from state to state?
Yes, there is a lot of variance in the value of gross domestic product from state to state and in the map colors.
7. Why is this an example of fractile interval categorization?
The range for each category is different, while the number of data points [states] in each category is equal.

Handout 3: Assessment—Answer Key (page 2 of 2)

Part II:

Directions: Refer to the GeoFRED map displayed in class to answer each question below.

8. In what units are the data reported in the map key?
Millions of dollars

9. How many categories of data are there?
Five

10. What is the interval for each category?
Calculate the difference between each pair of consecutive cutoff values

11. Which color represents the highest amount of gross domestic product by state?
The darkest color

12. Which color represents the lowest amount of gross domestic product by state?
The lightest color

13. Based on this map, does there appear to be much difference from state to state?
No, most states fall into the lowest category.

14. Why is this an example of equal interval categorization?
The range for each category is equal, while the number of data points (states) in each category is different.

Standards and Benchmarks

Voluntary National Content Standards in Economics

Standard 13

Income for most people is determined by the market value of the productive resources they sell. What workers earn depends, primarily, on the market value of what they produce and how productive they are.

- **Benchmarks: Grade 8**
 2. To earn income, people sell productive resources. These include their labor, capital, natural resources, and entrepreneurial talents.

National Geography Standards

Element One: The World in Spatial Terms

Standard 1

Globes and maps have been among the most ubiquitous tools for learning geography. They have been joined by aerial photographs, remotely sensed images, and geographic information systems. As technology makes them easier to make, maps and other geographic representations appear practically everywhere.